Princeton University
Department of Physics: Joseph Henry Laboratories
Jadwin Hall
Post Office Box 708
Princeton, New Jersey 08544-0708

June 21, 1989

Dr. A. Moroz
Institute of Physics
Czechoslovak Academy of Sciences
Na Slovance 2
18040 Prague 8
Czechoslovakia

Dear Dr. Moroz:

Thank you for your letter of 6 April 1989, and your preprint.

I am very interested in the questions you have raised but do not have complete answers. I have written an article in which I discuss related matters; a copy is enclosed. As you will see there, I asked myself the same question you asked me: Do the 't Hooft singularities actually occur in the ϕ_4^4 model? and proposed it as a subject for a rigorous treatment. However, I did not give a rigorous treatment in the paper.

Crutchfield promised to treat the double well dynamic oscillator in the second part of his paper, but so far as I know, it never appeared.

There is clearly a big difference between the double well case and the single well case. That is already evident from the zero dimensional result, where the integral

$$\int_{-\infty}^{\infty} e^{-\phi^2-\beta\phi^4} d\phi \qquad (1)$$

has a Borel summable perturbation series at $\beta = 0$, but

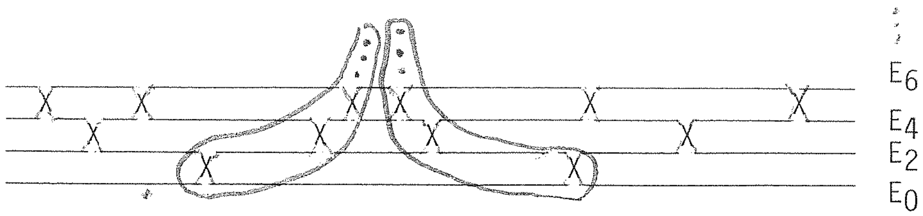
$$\int_{-\infty}^{\infty} e^{\phi^2-\beta\phi^4} d\phi \qquad (2)$$

does not, in general. I was surprized to find (see pp. 6, 7 of my paper) that after an instanton contribution is separates from (2) what is left has an asymptotic series in β.

I have been working with E. Bruening on the analogous problem in the anharmonic oscillator, but we still do not have an analogous result.

What is known for the double well anharmonic oscillator is that there are infinitely many branch points in the first and fourth quadrants of the β coupling constant plane. The best way to see how the sheets are connected is to follow Paul Shanley and look toward $-\infty$ along the real axis of the α plane. Then according to his numerical results one sees

Telephone: 609-452-4400 Telex: 499-3512



The horn of singularities consists of the branch points circled in red. The existence of the "extra" singularities not in the horn does not conflict with Barry Simon's statement that the singularities of E_n, for n fixed, must cluster to the axis. It is true for a trivial reason: they are invisible, being on other sheets. Thus, if the numerical evidence of Paul Shanley is not misleading, the eigenvalues of the double well anharmonic oscillator should have the right analyticity to have a Borel summable perturbation series. It would be nice to have a proof of this. Of course this does not clear up the situation for the Schwinger functions.

I notice that on pp. 6, 7 of your preprint you say you have a solution of the horn-shaped singularity problem, and a generalization of the Nevanluina theorem, which will appear in your next paper. Bruening and I have such a result but only for function having asymptotic series which satisfy conditions much stronger than those which are known to hold in ϕ_4^4. Can you treat the double well anharmonic oscillator with your method? Will it appear in your next preprint?

Sincerely,

A. S. Wightman

A. S. Wightman

ASW/lan